

# Dynamics of Two-Component Bose-Einstein Condensates Coupled with Environment

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We investigate the dynamics of an open Bose-Einstein condensate system consisting of two hyperfine states of the same atomic species which are coupled by tunable Raman laser. It is already suggested that the detuning between the laser frequency and transition frequency affect significantly on the dynamics of the pure condensate. Here we show that the detuning effect is suppressed by noise and dissipation caused by the environment. The increase of coherence and purity are also displayed for specific parameters. As a verification to the lowest-order approximation we derive the hierarchy of motion equations in the second-order approximation. It turns out that the former one can describe the dynamical evolution qualitatively for weak noise and dissipation and quantitatively for strong noise and dissipation.

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## I. INTRODUCTION

With the rapid experimental progress in manipulation of quantum gases multi-component atomic gases have now been an active research field in cold atomic physics. Since Myatt et al. first produced a binary mixture of condensates consisting of two hyperfine states ( $|F = 1, m_f = -1\rangle$  and  $|F = 2, m_f = 2\rangle$ ) of  $^{87}\text{Rb}$  [1], the two-component atomic bosons of  $^{87}\text{Rb}$  in  $|F = 1, m_f = -1\rangle$  and  $|F = 2, m_f = 1\rangle$  states received intensive study in experiments subsequently [2–7]. A variety of dynamical behaviors have been observed both in the Bose-Einstein condensate (BEC) [2–4] and in the noncondensed sample [5]. Meanwhile, theoretical works have predicted that the two-component Bose gas may exhibit exotic ground state and vortex structures [8–12] as well as interesting dynamical properties, such as the Rabi oscillation of population between the two states [2], nonlinear population dynamics [13] and quantum self-trapping [14].

In the two-component atomic gas, the two hyperfine states ( $|F = 1, m_f = -1\rangle$  and  $|F = 2, m_f = 1\rangle$ ) can be coupled by a two-photon transition which converts the  $^{87}\text{Rb}$  atom from one state to the other [2–4, 7]. This transition thus drives the population dynamics. In this case, the two-component system is analogous to the Bose gas trapped in the double-well and both of them have also been investigated intensively in recent years [14–16]. The dynamical properties of the condensate satisfy the time-dependent Gross-Pitaevskii (GP) theory because the Bose atoms condensed in the ground state and the system can be described by the macroscopic wavefunction.

However, under experimental conditions, the condensates should be regarded as an open system coupled with the environment since the condensate usually coexists with noncondensed thermal cloud and atom loss is also

unavoidable. In this situation, noise and dissipation may play important roles. Many theoretical works have discussed the noise and dissipation-induced effects in the open double-well condensates recently [17–20], and indicated that negligible changes in dynamical properties of the condensate appear, such as the decay of quantum self-trapping [17], decoherence [18] and dephasing [19, 20]. The dephasing phenomenon has even been observed in experiments [21]. However, sometimes dissipations may result in enhancement of the quantum effect. For example, a thermally enhanced quantum-oscillation was observed during the domain formation process in the ferromagnetic spin-1 condensate [22]. Li et al. also found that the maximum spin squeezing can be reached even in presence of particle losses [23]. Most recently, Witthaut et al. showed that the dissipation could lead to enhancement of coherence in the double-well BECs under specific conditions [24].

In this paper we will investigate the open two-component BECs. Although this system can be mapped into a double-well condensate model, it is of interest in its own right. For example, dynamics of this system can be controlled by tuning the laser frequency by which the two internal states of atoms are coupled [13, 14]. So our main purpose is to discuss the noise and dissipation-induced effects when the two components are coupled by the detuned laser. The Hamiltonian is expressed using the pseudo-angular-momentum operators and the dynamical properties is evaluated by a hierarchy of ordinary differential equations of Bloch vectors, which are formulated on the basis of master equation method [24].

The paper is organized as follows. Section II formulates equations of motion in the first-order approximation for an open two-component Bose condensate with noise and dissipation. Section III discusses dynamics of Bloch vectors for both the closed and open systems, respectively. Section IV is devoted to evolutions of the coherence and purity for certain given parameters. In Section V, we extend the motion equations to the second-order approximation and compare the results with those in the first order approximation. A summary is given in the last

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section.

## II. MODEL AND METHOD

The two-component BEC is described by the Hamiltonian of second quantization in the single mode approximation (natural unit  $\hbar = 1$  is used throughout the paper)

$$\hat{H} = \sum_{j=1,2} \hat{H}_j + \hat{H}_{int} + \hat{H}_f, \quad (1)$$

where

$$\hat{H}_j = \omega_j \hat{n}_j + \frac{g_j}{2} \hat{n}_j^2$$

with  $\omega_j = \int d^3r \phi_j^*(r) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_j(r) \right] \phi_j(r)$ . The interaction term between the two hyperfine states of atoms takes the form of

$$\hat{H}_{int} = g_{12} \hat{n}_1 \hat{n}_2,$$

and the transition term is

$$\hat{H}_f = G \left( \hat{a}_1^\dagger \hat{a}_2 e^{i\varphi(t)} + \hat{a}_2^\dagger \hat{a}_1 e^{-i\varphi(t)} \right).$$

Here  $g_j$  ( $j=1,2$ ) and  $g_{12}$  are the effective intra- and inter-components interaction constants that can be controlled experimentally by tuning the atomic  $s$ -wave scattering length with Feshbach resonance techniques. The transition probability between both components with a small detuning is denoted by the Josephson coupling strength  $G$ . In the rotating wave approximation the phase  $\varphi(t) = \Delta t$ , where  $\Delta$  is the detuning between coupling laser frequencies and transition frequency  $|\omega_1 - \omega_2|$  between two components.

The Hamiltonian can be reexpressed by introducing the Schwinger pseudo-angular-momentum operators defined as  $\hat{L}_x = \frac{1}{2} (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1)$ ,  $\hat{L}_y = \frac{1}{2i} (\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)$ ,  $\hat{L}_z = \frac{1}{2} (\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2)$  with Casimir invariant  $\hat{L}^2 = \frac{\hat{N}}{2} \left( \frac{\hat{N}}{2} + 1 \right)$ , where  $\hat{N} = \hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2$  is the total boson number operator.  $\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y$  and  $\hat{L}_z$  satisfy the usual angular momentum commutation relation:  $[\hat{L}_z, \hat{L}_\pm] = \pm \hat{L}_\pm$ , and  $[\hat{L}_+, \hat{L}_-] = 2\hat{L}_z$ . Therefore the Hamiltonian can be written as

$$\hat{H} = \omega_0 \hat{L}_z + q \hat{L}_z^2 + G \left( \hat{L}_+ e^{i\varphi(t)} + \hat{L}_- e^{-i\varphi(t)} \right) \quad (2)$$

with  $\omega_0 = \omega_1 - \omega_2 + (N-1)(g_1 - g_2)/2$  and  $q = (g_1 + g_2)/2 - g_{12}$ . In the Bose gas of  $|F=1, m_f=-1\rangle$  and  $|F=2, m_f=1\rangle$  states of  $^{87}\text{Rb}$ , the effective interaction constants are known to be in the proportion  $g_1 : g_{12} : g_2 = 1.03 : 1 : 0.97$  [2]. These parameters can be tuned by magnetic Feshbach resonance [25] so that they can be more different.

With the Schwinger representation the general solution of the Schrödinger equation  $i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \hat{H} |\psi(t)\rangle$  governing the dynamics of system can be obtained with a time-dependent unitary transformation [14] and when the system couples with the environment, its dynamics will reserve to the master equation

$$\dot{\hat{\rho}} = -i [\hat{H}, \hat{\rho}] + \mathcal{L}_p + \mathcal{L}_a, \quad (3)$$

where  $\hat{\rho}$  is the density operator,  $\mathcal{L}_p = -\frac{\gamma_p}{2} \sum_{j=1,2} (n_j^2 \rho + \rho n_j^2 - 2n_j \rho n_j)$  denotes the noise term and  $\mathcal{L}_a = -\frac{1}{2} \sum_{j=1,2} \gamma_{a_j} (\hat{a}_j^\dagger \hat{a}_j \rho + \rho \hat{a}_j^\dagger \hat{a}_j - 2\hat{a}_j \rho \hat{a}_j^\dagger)$  is the dissipation term. Accordingly,  $\gamma_p$  and  $\gamma_{a_j}$  describe, respectively, the strength of noise and the dissipation velocity for the component  $j$ . The one-time average of a system operator  $\hat{A}$  can be calculated by  $\langle \hat{A} \rangle = \text{tr} [\hat{A} \hat{\rho}(t)]$ . Here we define the single-particle Bloch vector and particle number as  $s_j(t) = 2\text{tr} [\hat{L}_j \hat{\rho}(t)]$  and  $n(t) = \text{tr} [(\hat{n}_1 + \hat{n}_2) \hat{\rho}(t)]$ , while the time derivatives of them are defined as  $\dot{s}_j(t) = 2\text{tr} [\hat{L}_j \dot{\hat{\rho}}(t)]$  and  $\dot{n}(t) = \text{tr} [\hat{N} \dot{\hat{\rho}}(t)]$ .

Insert the Hamiltonian  $\hat{H}$  into Eq. (3), we will find that the first-order operators  $\hat{L}_j$  depend not only on themselves, but also on the second-order moments  $\langle \hat{L}_i \hat{L}_j \rangle$ . Similarly, the time evolution of the second-order moments depends on third-order moments, and so on. In order to obtain a closed set of equations of motion, the hierarchy of equations must be truncated at some stage by approximating the  $N$ th order expectation value in terms of all lower-order moments. Here we take the first-order approximations, i.e.,  $\langle \hat{L}_i \hat{L}_j \rangle \approx \langle \hat{L}_i \rangle \langle \hat{L}_j \rangle$ , and thus the equations of motion for the Bloch vector take the formulation of

$$\begin{aligned} \dot{s}_x &= -\omega_0 s_y - q s_y s_z - 2G s_z \sin \varphi(t) - T_2^{-1} s_x, \\ \dot{s}_y &= \omega_0 s_x + q s_x s_z - 2G s_z \cos \varphi(t) - T_2^{-1} s_y, \\ \dot{s}_z &= 2G (s_x \sin \varphi(t) + s_y \cos \varphi(t)) - T_1^{-1} s_z - T_1^{-1} f_a n, \\ \dot{n} &= -T_1^{-1} n - T_1^{-1} f_a s_z. \end{aligned} \quad (4)$$

Here the damping parameters  $T_1$ ,  $T_2$  and the relative dissipation velocity  $f_a$  are expressed as

$$T_1^{-1} = \frac{1}{2} (\gamma_{a_1} + \gamma_{a_2}), T_2^{-1} = \gamma_p + T_1^{-1}, f_a = \frac{\gamma_{a_2} - \gamma_{a_1}}{\gamma_{a_2} + \gamma_{a_1}}.$$

The dissipation can be controlled artificially by shining a laser beam onto the condensates or be achieved by a forced radio frequency transition to an untrapped magnetic substate [26]. The relative dissipation rate  $f_a$  can be changed from 0 to 1 and we take a medium value 0.5 in the present paper if it is not given particularly when there exists atom loss.

In this work we will investigate the phase coherence between two species and purity of the system, both

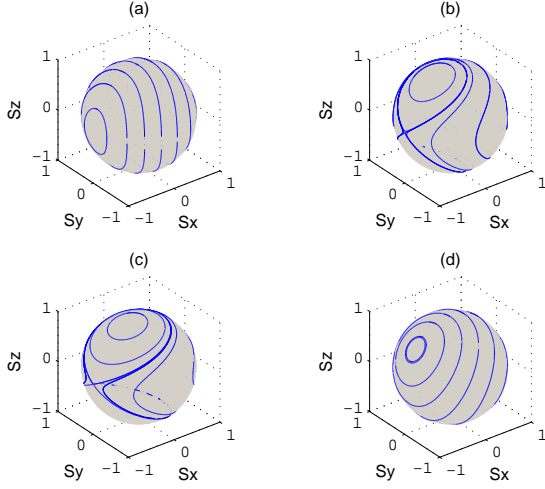


FIG. 1: The evolving Bloch vector for the closed system of  $N = 1000$  and  $\Delta = 0$ . (a)  $q=0.001$ ,  $\omega_0=0.0$ ; (b)  $q=0.005$ ,  $\omega_0=0.0$ ; (c)  $q=0.008$ ,  $\omega_0=0.0$ ; (d)  $q=0.001$ ,  $\omega_0=1.0$ .

of which are related with the Bloch vector by  $\alpha(t) = 2 \left| \langle \hat{a}_1^\dagger \hat{a}_2 \rangle \right| / \langle \hat{n}_1 + \hat{n}_2 \rangle = \sqrt{s_x^2 + s_y^2} / n$  and  $p(t) = |\mathbf{s}|^2 / n^2$  such that the dynamics of them can be investigated by solving Eq. (4) for the definite initial condition. For convenience in the following evaluation the initial conditions  $\mathbf{s}(0) = (s_x(0), s_y(0), s_z(0))$  will be taken as  $N(a, 0, \sqrt{1-a^2})$  with  $N$  being the initial particle number in the system.  $s_x(0) = 0$  and  $s_z(0) = N$  ( $a=0$ ) corresponds to the case that all atoms occupy in the same hyperfine state while  $s_x(0) = N$  and  $s_z(0) = 0$  ( $a=1$ ) corresponds to the case that atoms populate equally in two hyperfine states. In the following evaluation we will consider the system of  $N = 1000$ .

### III. DYNAMICS OF BLOCH VECTOR

In the present Schwinger representation all interesting physical quantities can be formulated with Bloch vector. For example, for particle number conserved system coherence is the module of the projection of Bloch vector on the  $s_x$ - $s_y$  plane, its purity is the module square of Bloch vector and  $s_z(t)$  corresponds to the population imbalance between two components. So the dynamical evolution of Bloch vector is worth to study very much. This section will focus on the evolving Bloch vector and the explicit investigation of coherence and purity will be given in the next section.

#### A. The Closed System

Firstly, we investigate the dynamics of the system without coupling with environment, i.e., there is not noise and particle dissipation. In Fig. 1 (a-c) we display the

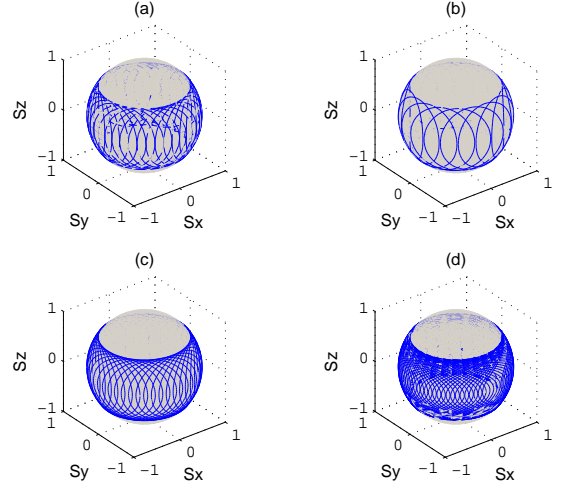


FIG. 2: The evolving Bloch vector for the closed system of  $N = 1000$  and  $\omega_0=0.0$ . The initial state is chosen as  $s_x(0) = 0.6N$ . (a)  $q=0.0$ ,  $\Delta=0.05$ ; (b)  $q=0.0$ ,  $\Delta=0.1$ ; (c)  $q=0.001$ ,  $\Delta=0.05$ ; (d)  $q=0.01$ ,  $\Delta=0.05$ .

Bloch vector for different nonlinear constants  $q$  for the case of detuning  $\Delta = 0$ . The Bloch vector evolves periodically in the surface of Bloch sphere and forms a closed orbital, while different initial condition corresponds to respective orbital.

For the weak nonlinear constants the orbitals behave as an almost perfect circular orbital parallel to the plane of  $s_x = 0$  and the fixed point belong to the line ( $s_y = s_z = 0$ ). That is to say, atoms shall oscillate periodically between these two hyperfine states and in each period the average occupation probability is equal in both states. With the increase of nonlinear term the original circle orbital shall deform and their centers moved from the position of  $s_z = 0$  ( $q = 0$ ) to the north pole of  $s_z = 1$ . The fixed points move to the upper sphere. For example,  $s_z(t)$  approaches to 1.0 for all time as  $q = 0.008$ . This is referred to macroscopic quantum self-trapping (MQST) that atoms will prefer to stay in the same hyperfine state instead that during an oscillation period atoms occupy in equal probability in both states. The transition between periodical oscillation and MQST corresponds to the orbital similar to lemniscates spreading in the surface of Bloch sphere.

It deserves to notice that in Ref. [7] those new formed two trajectories and fixed points  $F_{\pm}$  for the strong interaction correspond to  $\pm z_0$ , respectively, while in the present evaluation we only choose the positive  $z_0$ . In addition the transition from the periodical oscillation to MQST can be induced not only by the increased interaction but also the increased initial imbalances between two states.

The effect of  $\omega_0$  can be displayed by comparing Fig. 1a ( $\omega_0=0$ ) and Fig. 1d ( $\omega_0=1.0$ ). It is shown that its value shall change the orientation of the orbital plane. The increase of  $\omega_0$  leads to the turn to the north pole of the

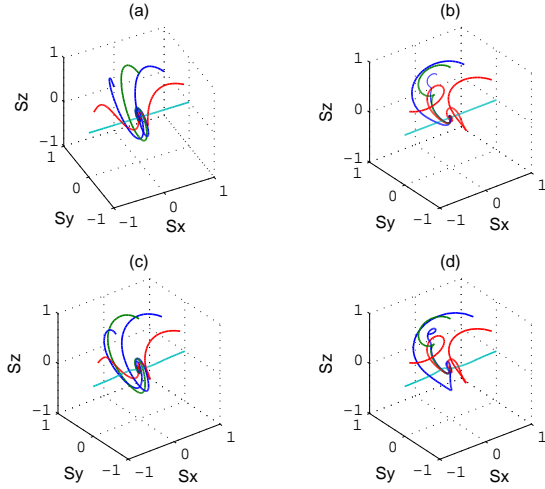


FIG. 3: The evolving Bloch vector for the system of  $N = 1000$  subject to noise for  $\omega_0=0.0$ ,  $\gamma_p=1.0$  and  $T_1^{-1}=0.0$ . (a)  $q=0.00$ ,  $\Delta=0.0$ ; (b)  $q=0.005$ ,  $\Delta=0.0$ ; (c)  $q=0.001$ ,  $\Delta=0.05$ ; (d)  $q=0.005$ ,  $\Delta=0.05$ .

orientation of orbital plane, which is also an expression of MQST. In a word, for the closed system of nondetuning MQST can exhibit by tuning  $\omega_0$ ,  $q$  and initial imbalance, whose orbital center deviate from  $s_z = 0$ .

In Fig. 2 we display the orbitals for the situation where there exist detune between the transition frequency and the coupling laser. In this case the Bloch vector will no longer evolve periodically and its path behave as a spiral line in the surface of Bloch sphere. With the time delay the path of Bloch vector wind each other and looks like a basket when the evolving time is long enough. By comparing Fig. 2a with Fig. 2b and Fig. 2c with Fig. 2d we found that the sparser the net grid, the bigger the detuning and the denser the net grid, the stronger the nonlinear constant. The denser track means that the evolution of Bloch vector and thus the dynamics of two-component BECs is more close to the pure periodicity in short time. In spite of the complicated orbital its projection on  $s_z$  axis is still periodical oscillation while the period is larger a little than the case without detune. The evaluation show that its period become longer although the atoms still transit periodically between two states and the transition point between delocalization and self-trapping also deviate slightly from the case without detune.

### B. The System Coupling With Environment

After investigating the dynamics of closed system, we focus on the system coupling with environment. The effects of noise and dissipation are shown in Fig. 3 and in Fig. 4, respectively. Here different orbitals in Bloch space correspond to different initial condition.

For the system subject to noise the module of Bloch vector will not be conserved rather decrease to zero in

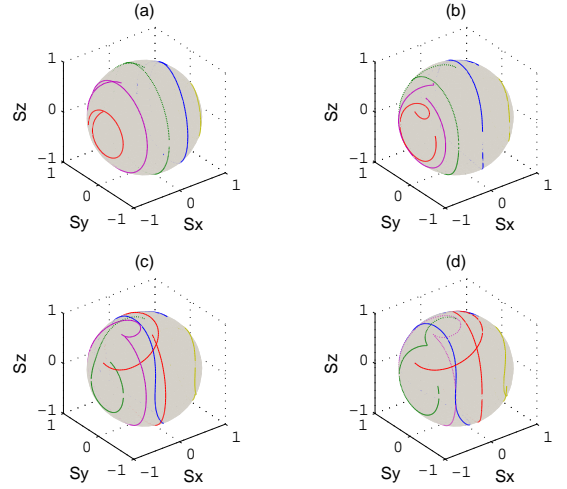


FIG. 4: The evolving Bloch vector for the system of  $N = 1000$  subject to dissipation for  $\omega_0=0.0$ ,  $\gamma_p=0.0$ ,  $\Delta=0.05$  and  $T_1^{-1}=1.0$ . (a)  $q=0.001$ ; (b)  $q=0.005$ ; (c)  $q=0.008$ ; (d)  $q=0.01$ .

a spiral during the dynamical evolution. It would not be on the surface of Bloch sphere rather shrink to the original point of coordinate frame even for very weak noise, whose strength only affect the evolving time to the original point. That is to say, when suffering noise instead of dissipation the atom number populated in two components would always tend to balance. In addition the purity of system tend to zero that corresponds to the module of Bloch vector. This will be discussed further in the later section. By comparing Fig. 3a with Fig. 3b and Fig. 3c with Fig. 3d we found that the stronger nonlinear interaction constant and large detuning shall induce that the Bloch vector run as an irregular orbital.

In Fig. 4 we plot the orbitals for the case that there are only dissipation instead of noise for different nonlinear interaction constants  $q$ . The Bloch vector shall be always on the surface of Bloch sphere but the orbitals become irregular and not closed. In this situation for different initial atom population in two components the final state shall be indefinite completely. With the increase of dissipation the atomic loss rate become faster and the lifetime of BECs become shorter and shorter. This means that along with the BECs oscillation between two hyperfine states aperiodically, more and more atoms escape from the system. As the interaction constant increases the shape of orbitals become more and more complicated.

Fig. 5 displays the evolution of Bloch vector for the system of 1000 atoms and  $q = 0.005$  subject to both noise and dissipation. Here  $f_a=0.5$  in Fig. 5 (a)-(e) and  $f_a=0.0$  in Fig. 5f. It is shown that for different initial states it will always evolve to the same point in the Bloch space, which correspond to the same final state, while the change of noise and dissipation shall determine its evolving path and dissipation velocity to the final state. In addition the joint effect on the BECs induces that the evolving path to the final state become more simple than

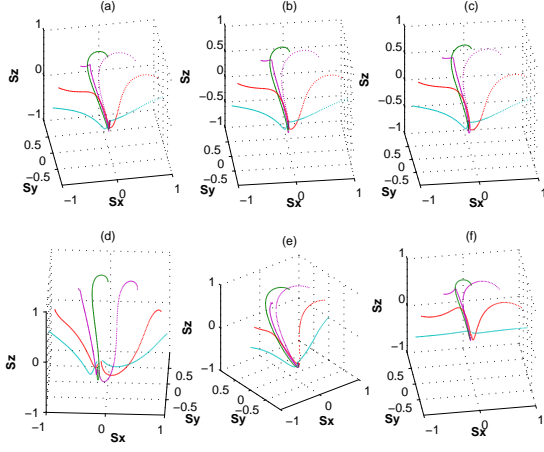


FIG. 5: The evolving Bloch vector for the system of  $N = 1000$  subject to both noise and dissipation for  $\omega_0=0.0$  and  $q=0.005$ . (a)-(c)  $\gamma_p=2.0$ ,  $T_1^{-1}=1.0$ ;  $\Delta=0.0$  (a),  $0.05$  (b),  $0.1$  (c). (d)  $\gamma_p=1.0$ ,  $T_1^{-1}=2.0$ ,  $\Delta=0.1$ . (e)  $\gamma_p=2.0$ ,  $T_1^{-1}=2.0$ ,  $\Delta=0.1$ . (f)  $\gamma_p=2.0$ ,  $T_1^{-1}=2.0$ ,  $\Delta=0.1$ ,  $f_a=0.0$ .

the system subject to only noise or dissipation. According to Fig. 5 (a)-(c) ( $\Delta=0.0, 0.05, 0.1$ ) the detuning between transition frequency and the frequency of laser induce no explicit effect. According to Fig. 5e and Fig. 5f, the relative dissipation rate of each components shall result in different final position in Bloch space, i.e., different final states.

To sum up, noise always induce that atoms populate in two components evenly and atom dissipation determine the lifetime of condensate. Both noise and dissipation shall result in the irregular evolving path of Bloch vector, i.e., the aperiodic dynamics. In this situation the system always arrive at the same final state and the evolving path shall be more simple compared with the case subject to only noise or dissipation.

#### IV. COHERENCE AND PURITY

In this section we will study the evolution of coherence and purity for different noise and dissipation.

In Fig. 6 the evolution of coherence is displayed for different initial conditions. It is shown that for a closed system the coherence will always oscillate periodically as long as the initial population difference in each component deviates from zero ( $a \neq 1$ ) while it will preserve the strongest coherence when the atoms initially populate equally at two components ( $a = 1$ ) (Fig. 6a). When the BECs suffer noise instead of dissipation the coherence will decrease oscillationly (Fig. 6b), and when there exist dissipation instead of noise the coherence still oscillates quasi-periodically (Fig. 6c). For the system exposed to both noise and dissipation the coherence show rich evolving dynamics determined by the initial populations (Fig. 6d). For the case of equal population initially

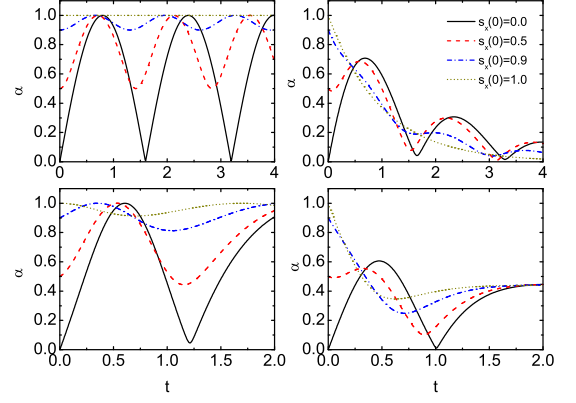


FIG. 6: The coherence dynamics for the system of  $q=0.001$ ,  $\omega_0=0$ ,  $G_0=1$ ,  $\Delta=0$  and  $f_a=0.5$ . (a)  $\gamma=0.0$ ,  $T_1^{-1}=0.0$ ; (b)  $\gamma=1.0$ ,  $T_1^{-1}=0.0$ ; (c)  $\gamma=0.0$ ,  $T_1^{-1}=2.0$ ; (d)  $\gamma=2.0$ ,  $T_1^{-1}=3.0$ .

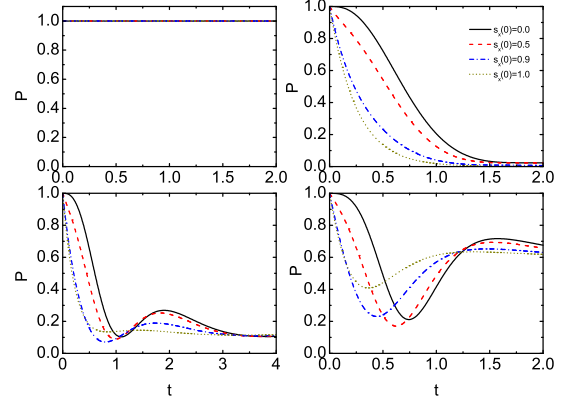


FIG. 7: The time evolution of purity for the system of  $q=0.001$ ,  $\omega_0=0$ ,  $G_0=1$ ,  $\Delta=0$  and  $f_a=0.5$ . (a)  $\gamma=0.0$ ,  $T_1^{-1}=2.0$ ; (b)  $\gamma=2.0$ ,  $T_1^{-1}=0.0$ ; (c)  $\gamma=2.0$ ,  $T_1^{-1}=1.0$ ; (d)  $\gamma=2.0$ ,  $T_1^{-1}=3.0$ .

( $a=1$ ) the coherence between two components decrease firstly and then increase, which is similar to the stochastic resonance. For the polarized case initially ( $a=0$ ) the coherence show a process of first increase from zero to the strongest value and then following a stochastic resonance. While for the initial population between these two limits ( $0 < a < 1$ ) coherence will show the evolving process of first increase or decrease and then following a stochastic resonance.

In short, particle loss do not temper with the coherence dynamics greatly and the obvious effect of noise is the reduction of the maximum coherence. So by tuning the strength of noise and dissipation rate we always can observe the process of coherence enhancement. The similar effect on the purity can also be seen (Fig. 7), where dissipation show no effect on purity.

The evolving purity is shown in Fig. 7. According to Fig. 7a ( $\gamma = 0$ ) and Fig. 7b ( $T_1^{-1} = 0$ ) the dissipation shall not bring about the change of purity as long

as the noise term is equal to zero and the noise shall result in the decrease of purity. By tuning the relative strength of noise and dissipation rate the purity increase again after rapid decrease. It turns out that for suitable parameter the purity can be enhanced to a very large value. For instance, it reach 0.7 for  $\gamma=2.0$ ,  $T_1^{-1}=3.0$ . In addition, the purity evolving do not show qualitative difference for different initial condition. It deserves to notice that the relative dissipation rate  $f_a$  play a critical role on whether the enhancement of coherence and purity can be displayed. Generally, the bigger  $f_a$  shall result in the larger coherence and purity so we take a medium value of  $f_a = 0.5$  here.

## V. SECOND-ORDER APPROXIMATION

In order to check the accuracy of the lowest-order approximation, we now take into account the second-order correction and compare the results from both cases. Up to the second-order approximation, the hierarchy equations are truncated by approximating the third-order expectation value  $\langle \hat{L}_i \hat{L}_j \hat{L}_k \rangle$  as the following:

$$\begin{aligned} \langle \hat{L}_i \hat{L}_j \hat{L}_k \rangle &= \langle \hat{L}_i \hat{L}_j \rangle \langle \hat{L}_k \rangle + \langle \hat{L}_i \rangle \langle \hat{L}_j \hat{L}_k \rangle \\ &+ \langle \hat{L}_i \hat{L}_k \rangle \langle \hat{L}_j \rangle - 2 \langle \hat{L}_i \rangle \langle \hat{L}_j \rangle \langle \hat{L}_k \rangle. \end{aligned}$$

Thus the hierarchy of motion equation shall be composed of the Bloch vector and the second-order moments  $\Delta_{ij} = 4(\langle \hat{L}_i \hat{L}_j + \hat{L}_j \hat{L}_i \rangle - 2 \langle \hat{L}_i \rangle \langle \hat{L}_j \rangle)$ . They can be formulated as below

$$\begin{aligned} \dot{s}_x &= -\omega_0 s_y - \frac{q}{2} (\Delta_{yz} + 2s_y s_z) - 2G s_z \sin \varphi(t) - T_2^{-1} s_x, \\ \dot{s}_y &= \omega_0 s_x + \frac{q}{2} (\Delta_{xz} + 2s_x s_z) - 2G s_z \cos \varphi(t) - T_2^{-1} s_y, \\ \dot{s}_z &= 2G (s_x \sin \varphi(t) + s_y \cos \varphi(t)) - T_1^{-1} s_z - T_1^{-1} f_a n, \\ \dot{n} &= -T_1^{-1} n - T_1^{-1} f_a s_z, \\ \dot{\Delta}_{xx} &= -2\omega_0 \Delta_{xy} - 4G \sin \varphi(t) \Delta_{xz} - 2q [\Delta_{xz} s_y + \Delta_{xy} s_z] \\ &\quad - 2T_2^{-1} \Delta_{xx} + 2(T_2^{-1} - T_1^{-1}) (\Delta_{yy} + 2s_y^2) + 2T_1^{-1} n, \\ \dot{\Delta}_{yy} &= 2\omega_0 \Delta_{xy} - 4G \cos \varphi(t) \Delta_{yz} + 2q (\Delta_{xy} s_z + \Delta_{yz} s_x) \\ &\quad - 2T_2^{-1} \Delta_{yy} + 2(T_2^{-1} - T_1^{-1}) (\Delta_{xx} + 2s_x^2) + 2T_1^{-1} n, \\ \dot{\Delta}_{zz} &= 4G (\cos \varphi(t) \Delta_{yz} + \sin \varphi(t) \Delta_{xz}) - 2T_1^{-1} (\Delta_{zz} - n), \\ \dot{\Delta}_{xy} &= \omega_0 (\Delta_{xx} - \Delta_{yy}) - 2G \cos \varphi(t) \Delta_{xz} - 2G \sin \varphi(t) \Delta_{yz} \\ &\quad + q (\Delta_{xx} s_z - \Delta_{yy} s_z + \Delta_{xz} s_x - \Delta_{yz} s_y) \\ &\quad - 2(2T_2^{-1} - T_1^{-1}) \Delta_{xy} - 4(T_2^{-1} - T_1^{-1}) s_x s_y, \\ \dot{\Delta}_{xz} &= -\omega_0 \Delta_{yz} - q \Delta_{yz} s_z - q \Delta_{zz} s_y + 2G \cos \varphi(t) \Delta_{xy} \\ &\quad - 2G \sin \varphi(t) [\Delta_{zz} - \Delta_{xx}] - [T_2^{-1} + T_1^{-1}] \Delta_{xz}, \\ \dot{\Delta}_{yz} &= \omega_0 \Delta_{xz} + q (\Delta_{xz} s_z + \Delta_{zz} s_x) + 2G \cos \varphi(t) (\Delta_{yy} \\ &\quad - \Delta_{zz}) + 2G \sin \varphi(t) \Delta_{xy} - [T_2^{-1} + T_1^{-1}] \Delta_{yz}. \end{aligned}$$

Fig. 8a and 8b display the coherence  $\alpha$  and purity  $P$  obtained based on both the lowest-order and second-order approximation. For strong noise and dissipation

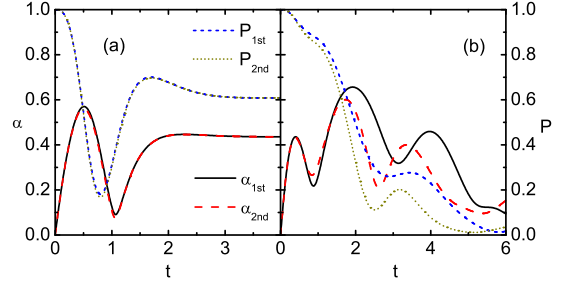


FIG. 8: Comparison between the lowest and second order mean-field approximation.  $q=0.001$ ,  $\omega_0=0$ ,  $G_0=1$ ,  $\Delta=0$ ,  $f_a=0.5$ . (a)  $\gamma=2.0$ ,  $T_1^{-1}=3.0$ ; (b)  $\gamma=0.5$ ,  $T_1^{-1}=0.5$ .

case, as shown in Fig. 8a, there is not obvious difference between the lowest-order and second-order results. For weak noise and dissipation case, although the second correction becomes more and more significant with the time, the tendencies of the curves are similar [see Fig. 8b]. Therefore, we conclude that the lowest order approximation are creditable qualitatively in the full parameter regime, especially in the presence of the noise and dissipation.

## VI. SUMMARY

In conclusion, we have investigated dynamics of the open two-component BEC in the Bloch representation. The two components correspond to two hyperfine states of the same specie of atom which are coupled by the detuned Raman laser. We have calculated the evolving Bloch vector, coherence and purity for both the closed system and open system.

For the closed system, when transition frequency match with Raman laser, the evolving Bloch vector form a closed orbital in the surface of Bloch sphere. This mean that all physical quantities for this system evolve periodically. As the nonlinear term becomes strong the MQST exhibits. In the detuning case, the orbital shall not be closed and wind into a basketlike path. In comparison with the non-detuning case, although atoms still transit between two states periodically, its transition period become longer. For the system subject to noise the atoms always evolve to a balance state that corresponds to the zero point in the Bloch space. In the situation of strong nonlinear term and large detuning the orbital of Bloch vector shall be irregular. For the system of dissipation, along with atoms oscillate aperiodically between two states more and more atoms escape from the condensates. The evolving Bloch vector behave as an irregular orbital in the surface of Bloch sphere. When the condensates subject to both noise and dissipation the system always evolve to the same final state although the evolving path might be different for different noise strength and dissipation rate. It deserves to notice that the detuning



between transition frequency and laser frequency change obviously the dynamics of the closed system, while the detuning effect is not displayed obviously in the presence of noise and dissipation.

We also studied the coherence and purity for both closed and open system. For the closed system the coherence always oscillates periodically. As subject to noise the coherence shall decay to zero during the period of evolution, while the particle loss induce no obvious effect on the coherence. By tuning the noise strength and dissipation rate the interesting effect of coherence enhancement exhibits. The investigation of purity show that the system preserve the maximum purity as long as it is not subject to noise even if the dissipation exists. In spite that the noise shall decrease the purity, it can be enhanced to a large value by tuning the noise strength and dissipation.

We discussed the second-order correction of the master equations. Comparison between the results from the lowest-order approximation with those from the second-order one has shown that the former can describe qualitatively the dynamics of both open and closed BECs and with the enhancement of coupling with environment its quantitative result shall be exact.

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